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A COUPLED DISCRETE-ELEMENT MODEL OF FLUID-SATURATED ROCK AND THE RESULTS OF STUDYING OF THE IMPACT OF A FLUID ON THE SHEAR STRENGTH OF A ROCK UNDER COMBINED COMPRESSION AND SHEAR

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Abstract. Within a discrete-element model of a porous permeable elastic-plastic rock, filled with a fluid, we have studied the shear strength of a fractured interface zone (a shear band) between blocks of a geological medium subject to compression and shear. Under these conditions, a fluid pore pressure is controlled by interplay of dilation of the elastic-plastic shear band and fluid transport between the blocks and the interface. We have found that the shear strength is a unique function of a combination of parameters, which includes viscosity of a fluid, permeability of the medium, shear rate and a characteristic size of the system. Based on the simulation results we have constructed the generalized binomial dependence of the shear strength of samples on the obtained combination of parameters.

1 INTRODUCTION

Inelastic deformation and stress relaxation in rocks are conditioned with nucleation and integration of fractures, accompanied with a dilation of a geological medium [1]. Blocks of a geological medium are separated with high-fragmented shear bands, which strength is significantly lower than strength of blocks [2]. In the result, deformation of rocks localizes predominantly within shear bands. Cracks, pores and voids inside a shear band form an interconnected system, which can be filled with a pore fluid in natural conditions. A yield stress and shear strength of a shear band depend on the effective stress $\sigma_{\alpha\beta}^{eff} = \sigma_{\alpha\beta} + \delta_{\alpha\beta}\chi P_{pore}$, where $\sigma_{\alpha\beta}$ is an applied external pressure, P_{pore} – pore fluid pressure, $0 < \chi \le 1$ – a material parameter, $\delta_{\alpha\beta}$ – the Kronecker symbol ($\alpha, \beta = x, y, z$) [3].

Dilation of a shear band under a plastic shearing leads to increasing of the volume of pores and cracks; in the results a fluid pore pressure P_{pore} decreases. A decrease of fluid pore pressure on walls of pores and cracks inhibits the stress relaxation processes, conditioned with a generation of new fractures and an integration of existing ones. This leads to an increasing of a macroscopic strength of a geological material. The mentioned effect has been called as dilatant hardening.

Evidently, in permeable media a value of pore fluid pressure is controlled both by a pore volume change and by a filtration motion of a fluid inside pores and cracks of a material. At that a local filtration rate depends on a permeability, a viscosity of a fluid and a gradient of pore pressure. Relation between a strain rate and a filtration rate determines a regime of deformation of a fluid-saturated material (drained or undrained conditions) [4]. Estimates of strength of a porous fluid-saturated material have been obtained by Biot for limiting cases of drained and undrained conditions [5][6]. The dependencies of a strength of a fluid-saturated materials on a dilation rate, filtration rate and strain rate were studied in detail in further works [7]-[10]. Particularly, it has been shown that the relation between rates of dilation (more precisely, pore volume increase under inelastic deformation) and filtration determines a value of fluid pore pressure in microcracks' tips, thereby controlling a development of faulting [11]-[12].

Despite the successes achieved in studying of mechanical properties of shear bands (including their shear strength), some keystone questions still remain unresolved. For instance, there was no a comprehensive study of strength of shear bands under combined compression and shear in the transition region between undrained and drained conditions (under partially drained conditions). It is clear that features of a shear strength in this region are determined not only by a shear rate and a permeability of a shear band itself, but to a large extent by a compression of a shear band and surrounding blocks of a material, fluid content, saturation and permeability of these blocks, as well as boundary conditions between a fragment of material under consideration and a host massif.

A direct study of deformation and strength of shear bands, appearing in samples under triaxial loading, represents a complicated problem. Therefore, a numerical simulation of a shear band under compression and shear seems to be a prospective approach to obtaining a detailed information about laws which determine strength of a shear band under complex loading conditions [13][14]. The goal of this work is to find a functional form of a dependence of shear band strength on the ratio of shear strain rate to filtration rate under conditions that correspond to rock massifs. The study has been carried out by means of a numerical simulation of a fluid-saturated elastic-plastic permeable medium with the Discrete Element Method (DEM).

2 DESCRIPTION OF A MODEL

During simulation of an elastic-plastic permeable medium, we use a decomposition of the considered problem into two ones: 1) a description of a mechanical behavior of a solid skeleton and 2) a simulation of a mass transfer of a fluid within a filtration volume (which represents a system of interconnected channels, pores, cracks etc.). Following the ideas of Biot [5][6], we assume that stress-strain state of a discrete element is directly connected with a change of volume of pores and pore pressure of a fluid in the "micropores" inside the element.

For simulation of a mechanical response of a material we have implemented the model of rock plasticity with non-associated flow law and von Mises yield criterion (the so-called Nikolaevsky model [15]). This model adequately describes a mechanical response of a wide class of brittle materials (geological materials, ceramics etc.) at different scales with taking into account of influence of lower-scale fractured structure. The Nikolaevsky model postulates a linear relationship between volume and shear deformation rates of plastic deformation with coefficient Λ named the dilation coefficient. We have adopted the Nikolaevsky model to the DEM with use of so called Wilkins algorithm [16]. In the framework of this algorithm a solution of elastic-plastic problem is reduced to a solution of an elastic problem in increments and following correction of potential forces between particles (discrete elements) in accordance with the requirements of Nikolaevsky model, applied to values of local pressure and stress deviator [16].

In the framework of the proposed approach a solution of an elastic problem represents a calculation of normal and tangential forces acting from discrete element *i* as a result of interaction with a discrete element *j*. The corresponding equations are formulated based on a generalized Hooke's law in hypoelastic form [17]:

$$\begin{cases} \Delta\sigma_{i(j)} = \Delta F_{i(j)}^{centr} \big/ S_{ij} = 2G_i \Delta\varepsilon_{i(j)} + \left(1 - \dfrac{2G_i}{K_i}\right)\Delta\bar{\sigma}_i^{mean} \\ \Delta\tau_{i(j)} = \Delta F_{i(j)}^{\tan g} \big/ S_{ij} = 2G_i \Delta\gamma_{i(j)} \end{cases} \tag{1}$$

where symbol Δ indicates an increment of corresponding parameter during a time step Δt of numerical scheme; $\sigma_{i(j)}$ and $\tau_{i(j)}$ – are specific values of pair-wise central $F_{i(j)}^{centr}$ and tangential $F_{i(j)}^{\tan g}$ components of reaction force of *i*-th discrete element to *j*-th neighbour; S_{ij} – a contact square; G_i and K_i – shear and bulk moduli, correspondingly; $\Delta\varepsilon_{i(j)}$ and $\Delta\gamma_{i(j)}$ – increments of normal and shear strain of element *i* in pair *i-j*; $\bar{\sigma}_i^{mean}$ – average volume stress in element *i* [17],[18].

A stress state of a porous solid skeleton, containing a system of interconnected pores, channels and cracks depends both on a porosity and geometry of pores and cracks and their spatial distribution [19]. In the absence of a pronounced orientation of cracks in a solid skeleton, the fluid pressure in a pore volume contributes only into a hydrostatic pressure in a solid skeleton (namely, into a hydrostatic tension). In this approximation the influence of a fluid in "micropores" can be taken into consideration by means of including of fluid pore pressure into a relation for a central force:

$$\Delta\sigma_{i(j)} = \Delta F_{i(j)}^{centr} \big/ S_{ij} = 2G_i \left(\Delta\varepsilon_{i(j)} - \frac{\Delta P_i^{fluid}}{K_i}\right) + \left(1 - \frac{2G_i}{K_i}\right)\Delta\bar{\sigma}_i^{mean} \tag{2}$$

where P_i^{fluid} – contribution of a fluid pore pressure (in "micropores") into a mean stress in a volume of discrete element *i*. Note that the equation (2) is equal to the Hooke's law in a model of linear poroelasticity. The value of P_i^{fluid} is linearly related with average pore pressure P_i^{pore} of a fluid in micropores of discrete element *i*:

$$P_i^{fluid} = a_i P_i^{pore} \tag{3}$$

where $a_i = 1 - K_i / K_{s,i}$. Here $K_{s,i}$ is a bulk modulus of non-porous grains of a solid skeleton of a discrete element *i*. After solution of the elastic problem for an element *i* at current time step, an achievement of the Mises–Schleicher yield criterion is checked, with explicit taking into account of a fluid pore pressure:

$$\Phi_i = \beta_i\left(\bar{\sigma}_i^{mean} + b_i P_i^{pore}\right) + \bar{\sigma}_i^{eq}\big/\sqrt{3} \geq Y_i \tag{4}$$

where Y_i is a shear yield stress of a material of element i, β_i is a coefficient of internal friction, $\bar{\sigma}_i^{eq}$ – von Mises stress, averaged over a volume of a discrete element i, b_i – dimensionless coefficient. A value of the coefficient b_i is determined by geometry of pores, channels and cracks in a solid skeleton. When a configuration of a pore volume allows formation of a uniform distribution of a hydrostatic pressure in a local volume of a solid skeleton, the value of b_i is suggested to be equal to unity [1],[2]. At that, new cracks are assumed to appear from existing micropores or cracks. In the opposite case, when microscopic structure of a solid skeleton provides a more complicated interconnection between a pore pressure and fracture generation, the value of b_i is usually less than unity and depends on a porosity and pore pressure. The lower boundary of b_i usually equals to initial porosity ϕ_0 of non-deformed material [1].

When the yield condition (4) is satisfied, the reduction of components of stress tensor in a volume of discrete element i to a yield surface is performed. In accordance with [17], the mentioned reduction can be performed by means of the following correction of specific normal and tangential forces of interaction between i-th element and j-th neighbor:

$$\begin{cases} \sigma'_{i(j)} = \left(\sigma_{i(j)} - \bar{\sigma}_i^{mean}\right) M_i + (\bar{\sigma}_i^{mean} - N_i) \\ \tau'_{i(j)} = \tau_{i(j)} M_i \end{cases} \tag{5}$$

where $\left(\sigma'_{i(j)}, \tau'_{i(j)}\right)$ – are reduced values of specific reaction forces; $M_i = 1 - \left(\sqrt{3}/\bar{\sigma}_i^{\text{int}}\right)\left(3G_i\left(\Phi_i - Y_i\right)/\left(K_i\Lambda_i\beta_i + 3G_i\right)\right)$ – coefficient of reduction of stress deviator; $N_i = K_i\Lambda_i\left(\Phi_i - Y_i\right)/\left(K_i\Lambda_i\beta_i + G_i\right)$ – correction to a local mean stress, calculated after solving an elastic problem; Λ_i – dilation coefficient of material of element i.

A volume of a solid skeleton and, correspondingly, a pore volume change under the influence of internal and external stresses. At that, a specific volume of pores ϕ (or so called "microscopic" porosity) can be defined as follows:

$$\phi = \left(V_{pore}^{elast} + V_{pore}^{plast}\right)\big/V_{elem} \tag{6}$$

where V_{pore}^{elast} is a part of pore volume, which develops due to elastic deformations of material; and V_{pore}^{plast} is a part of pore volume, that appears as a result of "quasi-plastic" deformation of a material, namely as a result of opening of microscopic pores, cracks and other defects because of dilation of a material. Note that in the framework of the developed model we don't take into account a compaction of pores, which is valid for low-porous materials. Elastic change of pore volume is determined by the relation of bulk moduli of porous solid skeleton K and of non-porous monolithic grains that constitute the solid skeleton K_s:

$$V_{pore}^{elast} = V_{elem}^{init}\left[\phi_0 + 3\sigma_{mean}\left(\frac{1}{K} - \frac{1}{K_s}\right) + 3P^{pore}\left(\frac{1}{K} - \frac{1+\phi}{K_s}\right)\right] \tag{7}$$

In turn, “inelastic” change of pore volume due to dilation of a material is given by the following relation:

$$V_{pore}^{plast} = V_{elem}^{init}\Omega_{plast} \tag{8}$$

where Ω_i^{elast} and Ω_i^{plast} represent elastic and inelastic parts of volume deformation of a discrete element, that are formally determined as follows:

$$\begin{cases} \Omega_i^{elast} = 3\left(\bar{\sigma}_i^{mean} + P_i^{fluid}\right)/K_i \\ \Omega_i^{plast} = \left(\varepsilon_i^{xx} + \varepsilon_i^{yy} + \varepsilon_i^{zz}\right) - \Omega_i^{elast} \end{cases} \tag{9}$$

Here $\varepsilon_i^{\alpha\alpha}$ are diagonal components of strain tensor in a volume of a discrete element i [17][18]. We use the modified fracture criterion of Drucker-Prager that takes into account a contribution of a local pore pressure of a fluid in the following way:

$$\sigma_{DP} = 0.5(\lambda+1)\sigma_{eq} + 1.5(\lambda-1)\left(\sigma^{mean} + bP^{pore}\right) = \sigma_c \tag{10}$$

where $\lambda = \sigma_c/\sigma_t$ is the relation between compressive (σ_c) and tensile (σ_t) strengths of a material, the coefficient b is the same as in equation (4).

In the framework of the developed model of a fluid filtration we use the following assumptions: 1) a fluid may occupy a pore volume completely or partially; 2) a fluid is compressible; 3) an adsorption of a fluid on internal walls of pores, capillary effects and the effect of adsorption reduction of strength (Rehbinder effect) are not taken into account; and 4) a statistical distribution of sizes of micropores is not taken into account. In the framework of the latter assumption the pore volume is completely described by the following two parameters: a value of open “microscopic” porosity ϕ and a characteristic diameter of filtration channel d_{ch}, which controls the rate of fluid filtration through a solid porous skeleton. An adequate choice of the value of d_{ch} allows correct description of a mass transfer of a fluid, despite simplicity of the assumptions given above.

A state of a compressible liquid in pores can be described by the following equation [20]:

$$\rho(P) = \rho_0\left(1 + (P - P_0)/K_{fl}\right) \tag{11}$$

where ρ and P are the current values of fluid density and pressure, ρ_0 and P_0 are the values of the density and pressure under atmospheric conditions, K_{fl} – bulk modulus of a fluid. When the fluid occupies a pore volume only partially, we assume the fluid pressure equals to the atmospheric pressure P_0. Neglecting the influence of gravity, the equation of filtration transfer of a fluid can be written in the following form [20]:

$$\phi\frac{\partial\rho}{\partial t} = K_{fl}\nabla\left[\frac{k}{\eta}\nabla\rho\right] \tag{12}$$

where η – fluid viscosity, k – coefficient of permeability of a solid skeleton that can be estimated as follows [20]:

$$k = \phi d_{ch}^2 . \tag{13}$$

Note that, in the framework of the used assumptions, there is no mass transfer between elements with fluid pressure $\rho \leq \rho_0$.

3 SIMULATION OF A SHEAR LOADING IN FLUID-SATURATED MEDIUM

We have considered a shear loading of an infinitely long sample with periodic boundary conditions in lateral direction (see fig. 1a). The sample consists of two linearly elastic blocks separated by an elastic-plastic shear band (interface). Pore volumes of permeable elastic blocks and elastic-plastic interface have been saturated with water under initial atmospheric pressure. The diagrams of uni-axial loading of materials of the blocks and the interface are given in fig. 1b. The considered sample was mounted between thin impermeable layers of material, to which an external loading was applied.

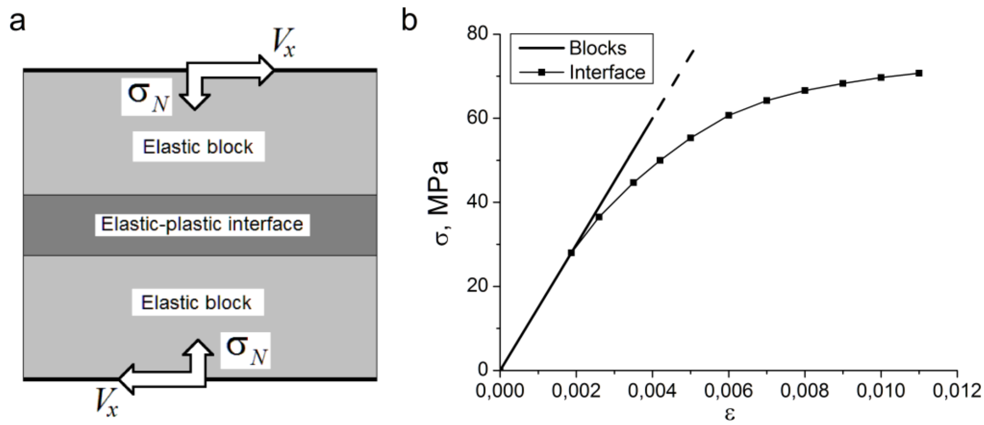

Figure 1: Scheme of loading (a); Diagrams of uni-axial loading of materials of the blocks and the interface (b).

The values of physical-mechanical parameters of the material are given in the table 1. The values of compressible and tensile strengths are given for the elastic-plastic interface, the elastic blocks are considered as indestructible. The total height of the sample was $L = 0.3$ m, the height of the interface was $L_0 = 0.03$ m.

The loading was performed in two stages. At the first stage an initial pre-loading with compression normal force F_N was performed. After that, we fixed the loading until fading of elastic waves in the sample. At the second stage a shear loading in lateral direction with the constant velocity V_x was applied until the sample fractures. At that, top and bottom layers were fixed in vertical direction.

We have found that under relatively small values of the normal pre-loading the fracture of the elastic-plastic interface occurs before a plastic deformation of the interface begins. At certain value of the normal pre-loading, the fracture of the interface goes after a plastic deformation begins and takes place at relatively high values of plastic deformations (see fig. 2). The latter demonstrates a "brittle-to-ductile" transition which occurs in real materials, in particular, in geological media.

Table 1: Physical-mechanical parameters of the solid skeleton

Parameter name	Value	Parameter name	Value
Open porosity of a skeleton ϕ	0.1	Compressive strength σ_c	70 MPa
Bulk modulus of a porous skeleton K	37.5 MPa	Tensile strength σ_t	23.3 MPa
Bulk modulus of monolithic grains K_s	107.5 MPa	Dilation coefficient Λ	0.36
Density of a porous skeleton	2000 kg/m^3	Internal friction coefficient β	0.57
Poisson ratio of a porous skeleton	0.3	Parameter b	0.1

The results, presented below, have been obtained in the "ductile" regime of fracture, i.e. when fracture occurs significantly after reaching a yield point. At that, the dependence of the shear strength on the normal confining pressure can be approximated with the following equation:

$$\tau_c \approx \tau_{c,0}\left(\sigma_N / \sigma_y\right)^{0.45} \tag{14}$$

where $\tau_{c,0}$ – is a scale factor, having the dimension of stress, and σ_y – is the yield strength.

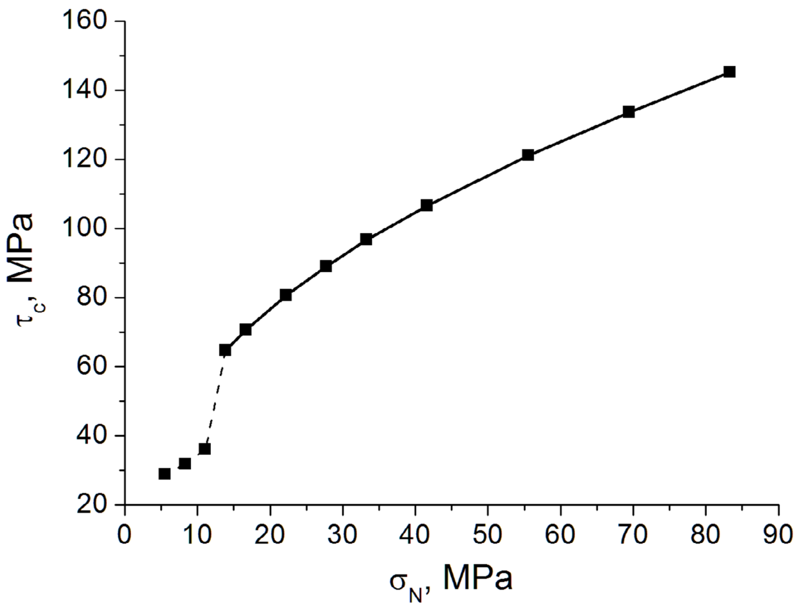

Figure 2: Shear strength of a non-permeable sample vs. different normal loads

The results of a parametric analysis of the problem have allowed us to obtain the governing combination of the parameters, which uniquely determines a value of the shear strength of the interface for a given value of the confining pressure:

$$A_{xy} = \mu \frac{L^2 \phi}{4k} \dot{\varepsilon}_{xy} \ . \tag{15}$$

The combination (15) includes viscosity of a fluid, porosity and permeability of the medium, shear rate and a height of the sample.

The dependency of the shear strength on the permeability of the material demonstrates an exponential decrease and further growth with an increase of the value of the permeability (see fig. 3). The dependencies of shear strength on permeability, obtained for different values of shear rate, viscosity, or height of the interface, can be reduced to a single dependence of shear strength on a unique controlling parameter (15). Note that the increase of the permeability corresponds to the decrease of the value of A_{xy} and vice versa.

The mentioned nonmonotonicity of the dependencies $\tau_c(A_{xy})$ has the following explanation. At relatively small values of permeability a liquid pressure in elastic-plastic interface rapidly decreases down to zero due to increase of the pore volume under dilation of elastic-plastic material. An outflow of the liquid from the regions, surrounding the interface, leads to slight decrease of the degree of constraint of the interface. In the result, the strength of the interface exponentially decreases with the increase of the permeability due to reduction of the mean stress in the interface.

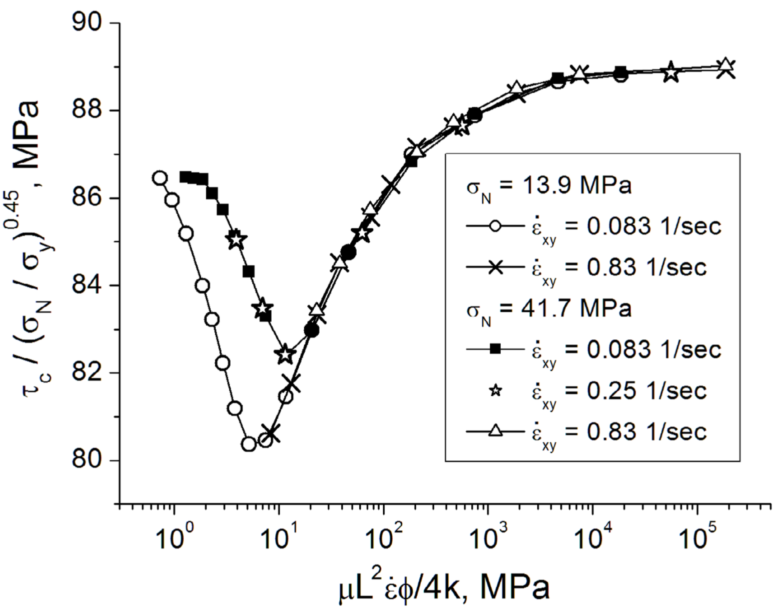

Figure 3: Normalized shear strength of a permeable sample for different shear rates and normal loads

At relatively high values of permeability a fluid pressure in the interface remains non-zero due to a rapid inflow of a fluid from the pore volume of the elastic blocks. The non-zero fluid pressure results in the decrease of the yield criterion in the accordance with the Mises-Schleicher criterion (4), at that, the dilation rate increases. This results in the increase of the degree of constraint and in the corresponding increase of the shear strength of the interface.

A competition of the processes, mentioned above, results in the occurrence of a minimal value of shear strength, where a rate of filtration is still not enough to provide a non-zero fluid pressure in the whole cross-section of elastic-plastic interface and, at the same time, it is enough to significantly decrease a pore pressure in the elastic blocks.

4 DISCUSSION AND CONCLUSIONS

Basing on the results shown above, we can conclude that shear strength of the interface depends on an interplay of the following processes: 1) increase of a mean stress in a medium under shear due to dilation of elastic-plastic shear band after reaching a yield point; 2) mass transfer of a fluid in a pore volume of the interface and the elastic blocks and 3) redistribution of the fluid in the sample due to the pressure gradient.

The observed effects together with the results of numerical simulations allowed us to suggest a generalized dependence of shear strength of an elastic-plastic interface on permeability and shear rate for a given normal load σ_N:

$$\tau_c = \sigma_0 + \sigma_1 \exp\left(-c_1 A_{xy}\right) + \frac{\sigma_2}{1+\left(c_2 A_{xy}\right)^{-p}} \tag{16}$$

where the value $(\sigma_0+\sigma_1)$ corresponds to the strength of impermeable water-filled sample (under “undrained” conditions) and the value $(\sigma_0+\sigma_2)$ represents the strength of a “dry” sample. Parameters c_1 and c_2 characterize the rate of change of exponential and sigmoidal branches of the dependence (16) with a change of permeability. Note that the values of σ_0, σ_1 and σ_2 are not constant but depend on a thickness of elastic-plastic interface, physical-mechanical parameters of a material and boundary conditions.

The first term of the dependence (16) describes the exponential decrease of strength of the interface due to the local decrease of the mean stress in the elastic blocks under outflow of a fluid into an excess pore volume in the interface. The second term characterizes the influence of filtration on the decrease of the yield criterion of the interface due to the growth of fluid pressure. This, in turn, leads to the increase of the degree of constraint of the interface.

The parameters of the obtained dependence (16) represent the combinations of the values of loading, width of the interface, physical-mechanical properties of the sample, including permeability, and physical mechanical properties of a fluid. A more detailed analysis of this dependence, including obtaining estimations for its unknown parameters is a subject of further research.

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